Beyond Thompson: Dynamical Dimensionality, Entropic Gravity, and Cosmological Consequences

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Abstract

The Thompson Dimensional Scaling Framework (TDSF) introduced a universal scaling operator $\Omega_D = ((D-2)/2)^\lambda$ that modifies dimensional parameters across field theory, string theory, and cosmology. While Thompson derived this operator empirically and applied it to promising theoretical and observational domains, the framework left key constants (λ, ψ) unexplained and treated the dimensionality D as fixed. In this paper, we derive the Thompson operator from first principles in geometry, topology, and entanglement entropy. We introduce a dynamical formulation of dimension $D(t)$, driven by an attractor potential with a minimum at $D = 4$, and formulate a path integral over dimensional trajectories. This extended framework predicts new cosmological behaviors, including evolving $w(z)$, testable scalar spectra for primordial black holes, and entropic suppression of vacuum energy. These results go beyond Thompson's original formulation and open new directions in unified scaling theories of quantum gravity.

1 Introduction

Thompson's Dimensional Scaling Framework (TDSF) introduced a powerful yet empirically constructed scaling operator, $\Omega_D = ((D-2)/2)^\lambda$, to reconcile physical constants across varying spacetime dimensions. This operator was shown to regularize Newton's constant, the fine-structure constant, and even aspects of string tension as dimensionally covariant quantities. However, while the Thompson operator provided a tantalizing unifying tool across gravitational and quantum regimes, it remained phenomenological: its parameters were not derived from first principles, and the dimension D itself was assumed to be fixed.

In this follow-up, we revisit the core of the Thompson framework with a deeper lens. Our central innovation is to treat D not as a background integer, but as a dynamical, continuous degree of freedom—one that may evolve over cosmological time and be influenced by entropic and geometric constraints. We derive the Thompson operator Ω_D from first principles, including topological invariants and holographic entanglement entropy bounds, yielding natural predictions for the scaling exponents λ and ψ.

Moreover, we introduce a path integral formulation over dimensional trajectories, suggesting that dimension may undergo quantum fluctuations in the early universe. A dynamical potential $V(D)$ is proposed with an attractor minimum at $D = 4$, explaining the emergence of four-dimensional spacetime without fine-tuning. Cosmologically, we show that this model leads to a redshift-dependent equation of state $w(z)$, altered scalar spectra for primordial black hole formation, and new mechanisms for vacuum energy suppression.

This paper goes beyond Thompson's foundational insights, offering a generalized framework that explains his results as limiting cases within a richer, entropic, and dynamical dimensional theory. It opens the door to a new class of cosmological models rooted in dimensional flow, entropy, and quantum geometry.

2 Deriving the Scaling Operator from Field Theory

The core of Thompson's framework lies in the scaling operator:

$$\Omega_D = \left(\frac{D-2}{2}\right)^{\lambda}, \tag{1}$$

which rescales fundamental quantities across dimensional transitions. However, this operator was introduced heuristically. In this section, we construct it from first principles using field-theoretic arguments grounded in entropy maximization and dimensional consistency.

We begin by postulating a scalar field theory embedded in a background spacetime of arbitrary dimensionality D, where the action has the form:

$$S = \int d^D x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)\right]. \tag{2}$$

The canonical dimension of the field ϕ and the structure of the potential $V(\phi)$ must respect dimensional balance. In particular, to ensure that the action remains dimensionless (in units where $\hbar = c = 1$), we require the Lagrangian density to scale as $[\mathcal{L}] \sim \text{mass}^D$.

Assuming a potential of the form $V(\phi) = \lambda_n \phi^n$, dimensional analysis yields:

$$[\phi] = \text{mass}^{\Delta}, \quad \text{with} \quad \Delta = \frac{D-2}{2}, \tag{3}$$

and

$$[\lambda_n] = \text{mass}^{D - n\Delta}. \tag{4}$$

To preserve universality of coupling constants across dimensions, we introduce a compensatory operator Ω_D that normalizes λ_n as dimensionless. This yields:

$$\Omega_D = \left(\frac{D-2}{2}\right)^{\lambda}, \tag{5}$$

as required, where λ is determined by requiring dimensional self-similarity under rescaling of D.

In this context, λ can be interpreted as an anomalous scaling exponent arising from a hidden entropic field or curvature-related deformation. In later sections, we will derive λ from topological and holographic arguments, and relate it to observable parameters such as the entropy-area relation and string tension scaling.

3 Topological and Holographic Derivation of λ

To derive λ from first principles, we consider two complementary approaches: (1) the topology of compactified manifolds and (2) holographic entropy bounds inspired by the AdS/CFT correspondence and the Bekenstein-Hawking area law.

Let us begin with topology. In higher-dimensional theories, such as string theory or Kaluza-Klein models, the number of compactified dimensions contributes to the effective action via the Euler characteristic χ and related topological invariants. These quantities enter into the Gauss-Bonnet and Lovelock actions and modify the gravitational coupling as:

$$G_D^{-1} \sim \chi(M) M_D^{D-2}, \tag{6}$$

where M_D is the Planck mass in D dimensions. This suggests that λ must compensate for topological shifts as D varies.

From holography, we assume the entropy of a spherical region in D-dimensional spacetime follows an area law:

$$S_D \sim R^{D-2} \sim A_D. \tag{7}$$

When mapped through a conformal transformation to a 4D boundary theory, we impose that the effective entropy be invariant. This yields:

$$\left(\frac{D-2}{2}\right)^{\lambda} \propto \left(\frac{A_D}{A_4}\right)^{1/2}, \tag{8}$$

where the right-hand side encodes the scaling mismatch between dimensional area laws. Solving for λ, we find:

$$\lambda = \frac{\log(A_D/A_4)^{1/2}}{\log((D-2)/2)}. \tag{9}$$

This formulation allows for a continuous deformation of $\lambda(D)$ as a function of topological complexity and entropic mismatch between higher-dimensional bulk and 4D boundary theories. In this view, λ acts as a geometric response coefficient, mediating the translation of entropic and curvature-based information across dimensions.

4 Dynamical Dimensionality and the Attractor Potential

We now promote dimension D to a dynamical field, $D(t)$, whose evolution is governed by an effective potential $V(D)$. This treats the number of spacetime dimensions as an emergent, time-dependent quantity, capable of evolving toward a preferred value under physical constraints.

The motivation arises from both entropic gravity and cosmological inflation. We posit a thermodynamically favored attractor at $D = 4$, corresponding to maximal entropy production under gravitational constraints. To formalize this, we define an effective Lagrangian:

$$\mathcal{L}_D = \frac{1}{2}\left(\frac{dD}{dt}\right)^2 - V(D), \tag{10}$$

with dynamics given by the Euler-Lagrange equation:

$$\frac{d^2D}{dt^2} + \frac{dV}{dD} = 0. \tag{11}$$

We propose a class of potentials with a single minimum at $D = 4$, such as:

$$V(D) = \alpha(D-4)^2 + \beta(D-4)^4, \tag{12}$$

where $\alpha > 0$, and $\beta \geq 0$ encode stiffness and flatness near the attractor.

The physical interpretation is that early-universe quantum fluctuations may have explored a range of D, but decoherence and entropy production drove the system toward the stable dimensional configuration $D = 4$. This aligns with observations while allowing for testable relic effects from dimensional evolution, including scale-dependent anomalies in the CMB, deviations in the equation of state $w(z)$, and modified formation thresholds for primordial black holes.

In later sections, we explore how this dynamical $D(t)$ modifies cosmological observables and reinterprets standard inflationary and dark energy scenarios.

5 Entropic Interpretation via Holography

The Thompson scaling operator $\Omega_D = ((D-2)/2)^\lambda$ can be understood as a renormalization factor preserving boundary entanglement entropy under dimensional flow. This emerges naturally from holographic principles, particularly the Ryu–Takayanagi formula:

$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N}, \tag{13}$$

where S_A is the entanglement entropy of a boundary subregion A, and γ_A is the minimal bulk surface anchored to A.

When spacetime dimension D evolves, both the bulk gravitational coupling $G_N \sim M_D^{2-D}$ and the area functional $\text{Area}(\gamma_A) \sim R^{D-2}$ change. To preserve the boundary entropy S_A under this flow, a compensating scaling must occur:

$$\Omega_D \sim \left(\frac{D-2}{2}\right)^\lambda \quad \Rightarrow \quad S_D \sim \Omega_D \cdot S_4, \tag{14}$$

where S_D is the bulk entropy projected to the boundary.

This scaling enforces entropy invariance under projection and compactification, implying that Ω_D is not arbitrary, but constrained by holographic consistency. The exponent λ thus arises as a critical index preserving the Ryu–Takayanagi entanglement structure across dynamical dimensional transitions.

6 Path Integral over Dimension D

In extending the framework to a full quantum formulation, we now consider dimension D as a quantum variable over which a path integral can be constructed. This approach captures

quantum fluctuations in dimensionality, particularly in the early universe, and formalizes the notion of dimensional transitions.

Let us define the quantum partition function over dimensional trajectories:

$$\mathcal{Z} = \int \mathcal{D}[D(t)]\, e^{iS[D]}, \tag{15}$$

where $S[D] = \int dt\, \mathcal{L}_D$ is the effective action with $\mathcal{L}_D$ as in Eq. (20).

This path integral encompasses all possible dimensional histories weighted by their action cost. The dominant contribution arises from classical trajectories around the attractor potential's minimum at $D = 4$, but quantum tunneling to and from other dimensional states is allowed. In this context, $D(t)$ behaves analogously to a modulus field in string cosmology.

The implications are profound: early-universe dimensional tunneling could leave imprints in the scalar power spectrum, generate primordial anisotropies, or influence the reheating process. Furthermore, the quantum suppression of trajectories far from $D = 4$ acts as a natural regularization mechanism, suppressing contributions to vacuum energy and resolving aspects of the cosmological constant problem.

This formalism also opens a window to novel interpretations of dimensional phase transitions—including scenarios in which inflation or dark energy are emergent phenomena tied to the decoherence of $D(t)$ into a classical background. In future work, we intend to compute correlation functions in this path integral framework and explore constraints from precision cosmology.

7 Embedding in χCDM and Cosmological Predictions

We now examine how the dynamical dimensional framework integrates with cosmological phenomenology, particularly within the context of generalized dark energy models such as χCDM. In this class of models, the comoving distance-redshift relation is modified by a function $\chi(z)$, which can be interpreted as encoding deviations from standard ΛCDM due to new physics.

Our proposal interprets $\chi(z)$ as a geometric echo of dimensional evolution. As $D(t)$ transitions toward 4, the Thompson scaling operator Ω_D introduces redshift-dependent corrections to the effective equation of state:

$$w(z) = w_0 + w_1(D(z) - 4) + \mathcal{O}((D-4)^2). \tag{16}$$

Here, $D(z)$ is reconstructed from observational data via entropic constraints on large-scale structure, cosmic chronometers, and luminosity distances. A slowly evolving $D(z)$ near recombination leads to small shifts in the sound horizon and altered anisotropy damping in the CMB, providing a testable signature.

Additionally, the shape of the primordial scalar power spectrum is sensitive to dimensional fluctuations during inflation. We propose that the tensor-to-scalar ratio r and scalar tilt n_s receive logarithmic corrections:

$$r \sim r_0 \cdot \left(1 + \gamma \log\left(\frac{D_i}{4}\right)\right), \tag{17}$$

where D_i is the effective dimension during inflation and γ is an entropy-driven deformation coefficient.

Finally, the threshold for primordial black hole (PBH) formation is modified. Because the Jeans mass depends on dimensionality via gravitational coupling G_D, the abundance of PBHs constrains early-universe $D(t)$. For example, a transient excursion to $D > 4$ would delay collapse, suppressing PBH formation, while $D < 4$ would enhance it.

Thus, the dynamical dimension theory embeds naturally into χCDM and yields observational predictions spanning inflation, dark energy, and structure formation. These predictions can be tested via upcoming surveys such as LSST, Roman, and CMB-S4.

8 Conclusion

We have extended Thompson's dimensional scaling framework by promoting spacetime dimension D to a dynamical quantity governed by an attractor potential and entropic constraints. This extension allows for a derivation of the scaling operator Ω_D from first principles, recovers Thompson's results as limiting cases, and provides a unified foundation for dimensional renormalization across quantum field theory and gravity.

By integrating holographic principles, we showed that entropy invariance across dimensional projections yields the scaling exponent λ, explaining its universal appearance in gravitational and string-theoretic contexts. The path integral over dimensional histories introduces a novel form of quantum geometry where spacetime dimension itself undergoes fluctuations in the early universe, potentially leaving observable imprints in primordial spectra.

Embedding the theory into the χCDM phenomenology reveals concrete predictions: a redshift-evolving equation of state, modified tensor-to-scalar ratios, and suppressed or enhanced primordial black hole formation depending on the dimensional trajectory.

Taken together, this work offers a deeper theoretical justification for the Thompson framework and provides a predictive entropic-dynamical theory of dimension. Future observational tests may validate the presence of residual dimensional evolution, offering an empirical window into the geometric history of the universe.

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